Introduction to Robotics

Configuration Space

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Path Planning: From Point Robots to Robots with Geometric Shapes

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- How can we plan a collision-free path when the robot has a geometric shape?

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- $\texttt{path} : [0, 1] \rightarrow Q_{free}$ is a continuous function with $\texttt{path}(0) = q_{init}$, $\texttt{path}(1) = q_{goal}$

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- How can the configuration be represented?

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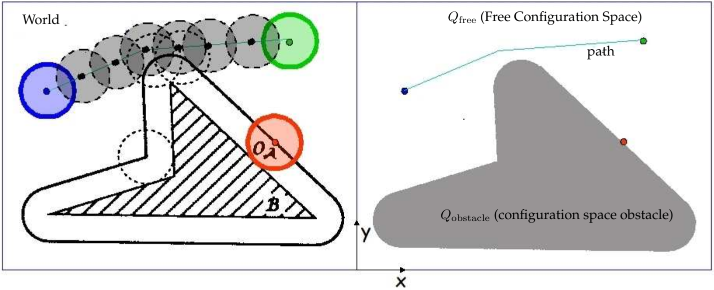

[Fig. courtesy of Latombe]

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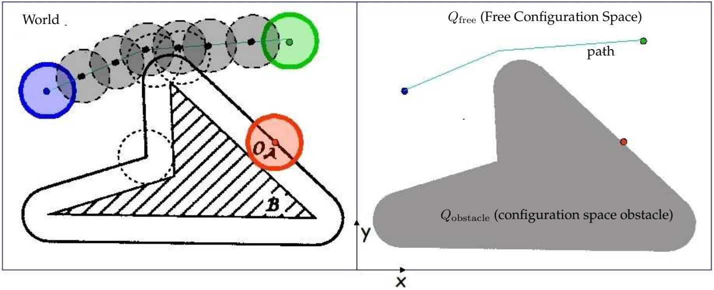

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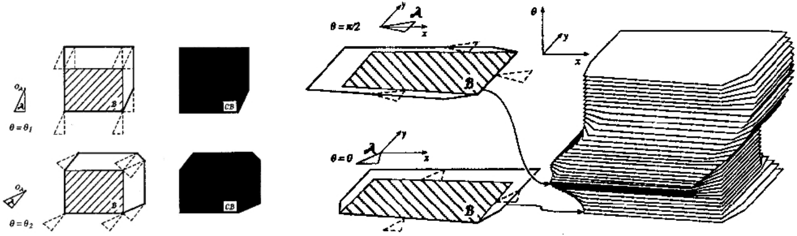

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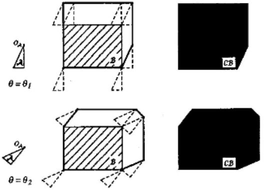

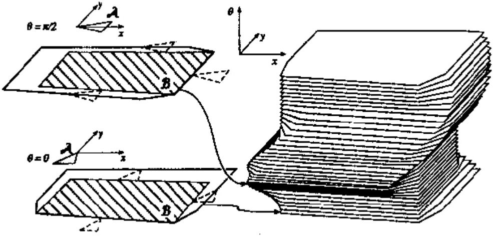

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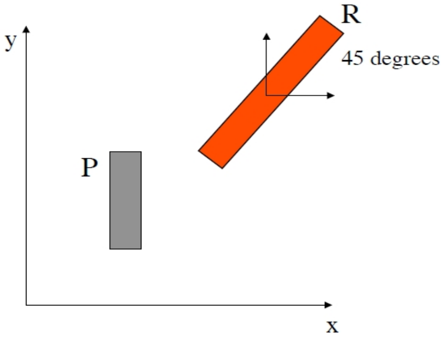
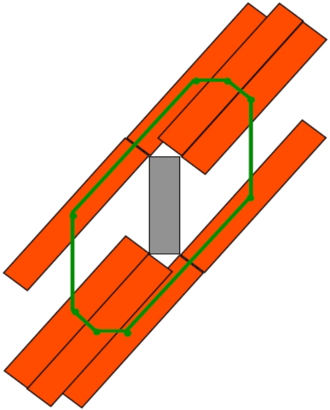
- How would you compute Q_{free}?

Taking the cros section of configuration space where robot is rotated at 45 degrees:





[Fig. courtesy of Choset, Dodds, Manocha]

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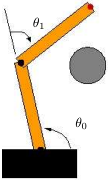

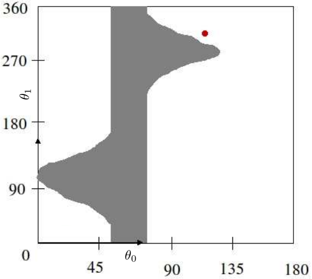

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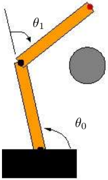

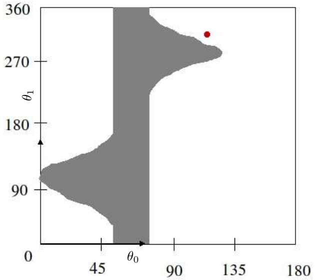

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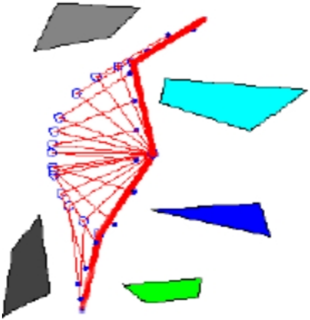
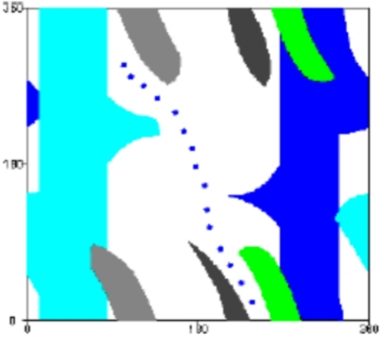
Two-link Path





Courtesy of Ken Goldberg

- The Minkowski sum of two sets A and B, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{a + b : a \in A, b \in B\}$$

- The Minkowski difference of two sets A and B, denoted by $A \ominus B$, is defined as

$$A \ominus B = \{a - b : a \in A, b \in B\}$$

How does it relate to path planning?

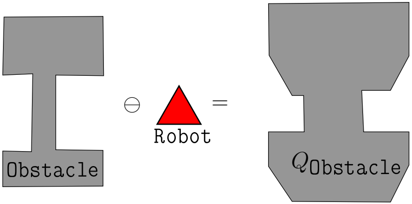
- Recall the definition of the configuration-space obstacle

$$Q_{\texttt{Obstacle}} = \{q : q \in Q \textit{ and } \texttt{Robot}(q) \cap \texttt{Obstacle} \neq \emptyset\}$$

(set of all robot configurations that collide with the obstacle)

- Classical result shown by Lozano-Perez and Wesley 1979

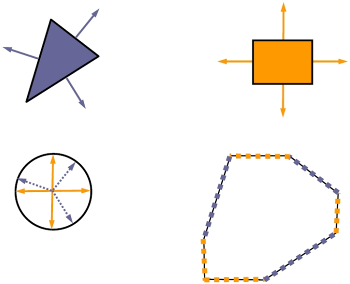
$$\text{for polygons and polyhedra} : Q_{\texttt{Obstacle}} = \texttt{Obstacle} \ominus \texttt{Robot}$$



[Fig. courtesy of Manocha]

Properties of Minkowski Sums

- Minkowski sum of two convex sets is convex
- Minkowski sum of two convex polygons A and B with m and n vertices ...
  - ... is a convex polygon with $m + n$ vertices
  - ... vertices of $A \oplus B$ are "sums" of vertices of A and B
  - ... $A \oplus B$ can be computed in linear time and space $O(n + m)$



[Fig. courtesy of Manocha]

Algorithm

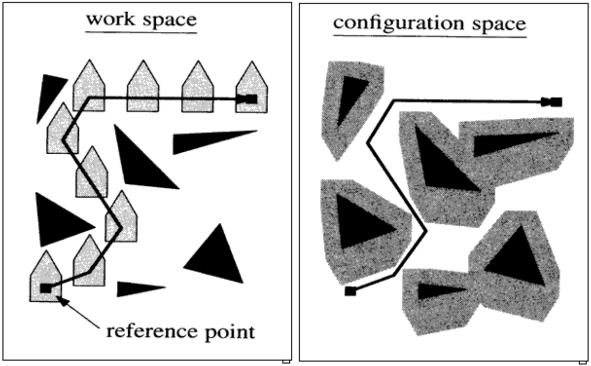
- sort edges according to angle between x-axis and edge normal
- let the sorted edges be $e_1, e_2, \ldots, e_{n+m}$
- attach edges one after the other so that edge e_{i+1} starts where edge e_i ends

- Minkowski sum for nonconvex polygons
  - Decompose into convex polygons (e.g., triangles, trapezoids)
  - Compute the minkowski sums of the convex polygons and take their union
  - Complexity: $O(n^2 m^2)$ (4-th order polynomial)
- 3D Minkowski sums: [convex: $O(nm)$ complexity] [nonconvex: $O(n^3 m^3)$ complexity]

Path Planning: From Point Robots to Robots with Geometric Shapes

- We have seen path-planning algorithms when a robot is a point
- How can we plan a collision-free path when the robot has a geometric shape?

... a key concept in path planning is the notion of a *configuration space*



- reduce robot to a point in the configuration space
- compute configuration-space obstacles (difficult to do in general)
- search for a path for the point robot in the free configuration space

Why study it?

- Extend results from one configuration space to another
- Design specialized algorithms that take advantage of certain topologies

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- Mathematical mechanisms for talking about topology: homeomorphism/diffeomorphism

$f : X \rightarrow Y$ is called a homeomorphism iff

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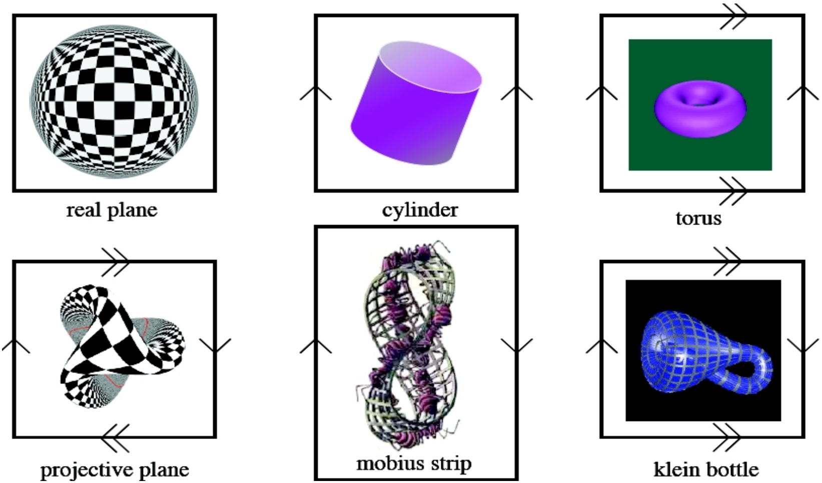
- A manifold is path-connected if there is a path between any two points



[Fig. courtesy of Choset, Dodds, Manocha]